International Journal of Scientific Research Updates

Journal homepage: https://orionjournals.com/ijsru/

ISSN: 2783-0160 (Online)

ORION SCHOLAR JOURNALS

(REVIEW ARTICLE)

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# Second type second order slope rotatable designs utilizing symmetrical unequal block arrangements with two unequal block sizes

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International Journal of Scientific Research Updates, 2024, 08(01), 054–064

Publication history: Received on 12 June 2024; revised on 24 July 2024; accepted on 27 July 2024

Article DOI: https://doi.org/10.53430/ijsru.2024.8.1.0047

## Abstract

Kim and Ko developed second order slope rotatable designs (SOSRD) of second type utilizing central composite designs (CCD), in which two numbers $(a_1, a_2)$ are used to represent the positions of star points. In this paper, we suggest second type SOSRD utilizing symmetrical unequal block arrangements (SUBA) with two unequal block sizes. In some cases, the suggested method may develop designs containing fewer design points than second type SOSRD obtained utilizing CCD, pairwise balanced designs (PBD) and balanced incomplete block designs (BIBD). The first order partial derivative's for estimated second order response variance for factors $6 \le v \le 15$ is also obtained.

**Keywords:** Central composite designs; Balanced incomplete block designs; Pairwise balanced designs; Symmetrical unequal block arrangements with two unequal block sizes; Second type second order slope rotatable designs;

## 1 Introduction

A response surfaces are set of statistical and mathematical models used to analyze the issues when several explanatory variables have an impact on a response variable. Box and Hunter [1] suggested the rotatability property for exploring response surface models and developed rotatable central composite designs (CCD). Das and Narasimham [6] studied second order rotatable designs (SORD) utilizing balanced incomplete block designs (BIBD). Raghavarao [13] developed SORD utilizing symmetrical unequal block arrangements (SUBA) with two unequal block sizes. Tyagi [24] developed SORD utilizing pairwise balanced designs (PBD). Kim [10] developed a rotatable CCD where in the two digits $(a_1, a_2)$ are used to denote the star points for $2 \le v \le 8$ (v: factors). Victorbabu [26] studied SORD utilizing a pair of partially balanced incomplete block designs (PBIBD). Victorbabu and Vasundharadevi [42] developed modified quadratic response surface models utilizing BIBD. Victorbabu [30] constructed modified quadratic response surface models, rotatable designs and rotatable designs with equi spaced doses. Victorbabu and Surekha [40, 41] developed rotatability measure for quadratic response surface models utilizing incomplete block designs (IBD) and BIBD respectively. Rajyalakshmi and Victorbabu [16] developed SORD under a tridiagonal correlated structure of errors utilizing BIBD. Jyostna and Victorbabu [8] constructed modified rotatability measure for a quadratic polynomial utilizing BIBD. Jyostna et al. [9] studied modified rotatability measure for quadratic polynomial models utilizing CCD. Chiranjeevi et al. [5] developed second type SORD utilizing CCD for $9 \le v \le 17$. Chiranjeevi and Victorbabu [2, 3, 4] constructed second type SORD utilizing BIBD, PBD and SUBA with two unequal block sizes respectively.

Slope rotatable central composite designs (SRCCD) were first developed by Hader and Park [7]. Victorbabu and Narasimham [32, 33, 34] further studied second order slope rotatable designs (SOSRD) utilizing BIBD, pair of IBD and

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PBD respectively. Park and Kim [12] studied a measure that helps us to evaluate the level of slope-rotatability for specific polynomial models. Victorbabu [25] constructed SOSRD utilizing SUBA with two unequal block sizes. Specifically, Kim and Ko [11] introduced the second type slope rotatability of CCD for the factors 2≤v≤5 by taking $n_a=1$ ('$n_a$' denotes the number of replications of axial points), where in the two numbers $(a_1, a_2)$ represent the positions of the star points. Victorbabu [27, 28] introduced modified SOSRD utilizing CCD, BIBD respectively. A review was proposed by Victorbabu [29] on SOSRD. Victorbabu and Surekha [36, 37, 38, 39] constructed SOSRD measure utilizing CCD, BIBD, PBD and SUBA with two unequal block sizes. Rajyalakshmi and Victorbabu [17] developed SOSRD under tri-diagonal correlated structure of errors utilizing BIBD. Rajyalakshmi et al. [18] constructed SOSRD under intra-class correlated errors utilizing PBD. Sulochana and Victorbabu [22, 23] constructed SOSRD under intra-class correlated structure of errors utilizing PBIBD and SOSRD under tri-diagonal correlation structure of errors utilizing a pair of IBD respectively. Victorbabu and Jyostna [31] constructed modified slope rotatability measure for quadratic polynomial models. Ravikumar and Victorbabu [19] extended the work of Kim & Ko [11] and constructed second type SOSRD utilizing CCD for 6≤v≤17 by taking $n_a=1$. Ravikumar and Victorbabu [20, 21] studied SRCCD second type for 2≤v≤17 with $2 \le n_a \le 4$ and SOSRD second type using PBD respectively. Victorbabu and Ravikumar [35] studied second type SOSRD using BIBD.

In this study, we suggest second type SOSRD utilizing SUBA with two unequal block sizes. It is find this suggested procedure sometimes results in an second type SOSRD with fewer design points than the SOSRD second type obtained through CCD of Kim and Ko [11], Ravikumar and Victorbabu [19], PBD of Ravikumar and Victorbabu [21] and BIBD of Victorbabu and Ravikumar [35] respectively.

## 2 Conditions for SOSRD

A general quadratic polynomial model $D=((X_{sw}))$ for fitting

$$Y_w=\beta_0+\sum_{s=1}^{v}\beta_s X_{sw}+\sum_{s=1}^{v}\beta_{ss}X_{sw}^2+\sum_{s<t}\sum\beta_{st}X_{sw}X_{tw}+\varepsilon_w \quad \text{............(2.1)}$$

In equation (2.1) $X_{sw}$ indicates the level of s[th] factor in the w[th] run (w=1,2,...,N) of the experiment and $\boldsymbol{\varepsilon}_w$'s uncorrelated random errors with mean '0' and variance $\sigma^2$. Then D is referred to be SOSRD if the $V\left(\frac{\partial \hat{Y}}{\partial X_s}\right)$ with regard to every explanatory variable $X_s$ is a function of the distance $\left(d^2=\sum_s X_s^2\right)$ of the point $(X_{1w}, X_{2w},...,X_{vw})$ from the origin (center) of the design.

The general conditions for SOSRD are given below (cf. [1], [7], [32]).

All moments of odd order are '0'. In simple terms when minimum of one odd power X equals zero. i.e;

A. $$\sum X_{sw}=0,\ \sum X_{sw}X_{tw}=0,\ \sum X_{sw}X_{tw}^2=0,\ \sum X_{sw}^3=0,\ \sum X_{sw}X_{tw}X_{uw}=0,$$
$$\sum X_{sw}X_{tw}X_{uw}^2=0,\ \sum X_{sw}X_{tw}^3=0,\ \sum X_{sw}X_{tw}X_{uw}X_{lw}=0,\ \text{etc. for } s\neq t\neq u\neq l;$$

B.(1) $$\sum X_{sw}^2=\text{constant}=N\delta_2$$

(2) $$\sum X_{sw}^4=\text{constant}=cN\delta_4,\ \forall\ s$$

C. $$\sum X_{sw}^2X_{tw}^2=\text{constant}=N\delta_4,\ \forall\ s\neq t \quad \text{............(2.2)}$$

Where c, $\delta_2$ and $\delta_4$ are constants.

The estimated parameters of variances and covariances are

$$\mathrm{Var}\left(\hat{\beta}_0\right)=\frac{\delta_4(c+v-1)\sigma^2}{N[\delta_4(c+v-1)-v\delta_2^2]}$$

$$\mathrm{Var}\left(\hat{\beta}_s\right)=\frac{\sigma^2}{N\delta_2}$$

$$\mathrm{Var}\left(\hat{\beta}_{st}\right)=\frac{\sigma^2}{N\delta_4}$$

$$\mathrm{Var}\left(\hat{\beta}_{ss}\right)=\frac{\sigma^2}{(c-1)N\delta_4}\left[\frac{(c+v-2)\delta_4-\delta_2^2(v-1)}{(c+v-1)\delta_4-\delta_2^2 v}\right]$$

$$\mathrm{Cov}\left(\hat{\beta}_0,\hat{\beta}_{ss}\right)=\frac{\sigma^2}{N}\left[\frac{-\delta_2}{[(c+v-1)\delta_4-\delta_2^2 v]}\right]$$

$$\mathrm{Cov}\left(\hat{\beta}_{ss},\hat{\beta}_{tt}\right)=\frac{\sigma^2}{(c-1)N\delta_4}\left[\frac{\left(\delta_2^2-\delta_4\right)}{[(c+v-1)\delta_4-\delta_2^2 v]}\right] \text{............(2.3)}$$

and the remaining covariances vanish.

D. $\frac{\delta_4}{\delta_2^2} > \frac{v}{c+v-1}$ ( Non-singularity condition )...........................(2.4)

From (2.1), we have

$$\frac{\partial\hat{Y}}{\partial X_s}=\hat{\beta}_s+2\hat{\beta}_{ss}X_{sw}+\sum_{s\neq t}\hat{\beta}_{st}X_{tw}$$

$$\mathrm{Var}\left(\frac{\partial\hat{Y}}{\partial X_s}\right)=\mathrm{Var}\left(\hat{\beta}_s\right)+4X_{sw}^2\mathrm{Var}\left(\hat{\beta}_{ss}\right)+\sum_{t\neq s}X_{tw}^2\mathrm{Var}\left(\hat{\beta}_{st}\right) \text{..............(2.5)}$$

The criteria for R.H.S. of (2.5) is to be a function of $d^2=\sum_{s=1}^{v}X_s^2$ alone (for slope rotatability) is

$$4\mathrm{Var}\left(\hat{\beta}_{ss}\right)=\mathrm{Var}\left(\hat{\beta}_{st}\right) \quad \text{(cf. [7]) ....................(2.6)}$$

Simplifying (2.6) using (2.3), we get

E. $\left[v(5-c)-(c-3)^2\right]\delta_4+\left[v\left(c-5\right)+4\right]\delta_2^2=0$ [cf. [32]]...................(2.7)

Therefore A, B, C of (2.2), (2.4) and (2.7) suggest a set of condition for slope rotatability in any general second-degree model. (cf. [7], [32]).

On solving equation (2.5), we get $\mathrm{Var}\left(\frac{\partial\hat{Y}}{\partial X_s}\right)=\frac{1}{N}\left[\frac{\delta_4+\delta_2 d^2}{\delta_2\delta_4}\right]\sigma^2$ ...........(2.8)

## 3 New second type SOSRD construction method utilizing SUBA with Two Unequal Block Sizes

Kim [10] developed second type rotatable CCD, in which the two digits $(a_1, a_2)$ are used to denote the star points for 2≤v≤8. Chiranjeevi et al. [5] extended the Kim [10] results and developed second type SORD utilizing CCD for 9≤v≤17. Chiranjeevi and Victorbabu [2, 3, 4] developed second type SORD utilizing BIBD, PBD and SUBA two unequal block sizes respectively. Specifically, Kim & Ko [11] developed slope rotatability of CCD of second type for 2≤v≤5 by taking $n_a = 1$. Ravikumar and Victorbabu [19] extended the results of Kim and Ko [11] and constructed SOSRD second type utilizing CCD for 6≤v≤17 by taking $n_a = 1$. Ravikumar and Victorbabu [20, 21] studied SRCCD second type for 2≤v≤17 with $2 \le n_a \le 4$ and SOSRD second type using PBD respectively. Victorbabu and Ravikumar [35] developed SOSRD second type using BIBD.

The design plan of second type SOSRD utilizing SUBA with two unequal block sizes is shown in Theorem 3.1. The v treatments are arranged in m blocks in which $m_1$ and $m_2$ blocks of sizes $k_1$ and $k_2$ respectively, $\left[k = \sup.(k_1, k_2), m = m_1 + m_2\right]$ is referred to as SUBA two unequal block sizes, if (cf. [15])

(a) Each treatment takes place in $\frac{m_i k_i}{v}$ blocks of size $k_i$, (i=1,2)

(b) Each set of first associate treatments takes place concurrently in p and $(\lambda\text{-}p)$ blocks of sizes $k_1$ and $k_2$ respectively, while each set of second associate treatments takes place concurrently in $\lambda$ blocks of size $k_2$.

Each treatment from (a) takes place in $\left(\frac{m_1 k_1}{v}\right) + \left(\frac{m_2 k_2}{v}\right) = r$ blocks among all.

Let $(v, m, r, k_1, k_2, m_1, m_2, \lambda)$ indicates the parameters of SUBA with two unequal block sizes, $2^{t(k)}$ indicate the fractional replicate of $2^k$ in -1 or +1 levels, in which there is no interaction is confounded with fewer than five factors. $n_0$ represents the central points.

Let $\left[1-(v, m, r, k_1, k_2, m_1, m_2, \lambda)\right]$ indicate the design points produced from the transposed incidence matrix of the SUBA with two unequal block sizes. Let $\left[1-(v, m, r, k_1, k_2, m_1, m_2, \lambda)\right]2^{t(k)}$ are the $m2^{t(k)}$ design points produced from SUBA with two unequal block sizes by multiplication (cf. [6]). We employ the extra set of points like $(\pm a_1, 0, \ldots, 0), (0, \pm a_1, 0, \ldots, 0), \ldots, (0, 0, \ldots, \pm a_1)$; $(\pm a_2, 0, \ldots, 0), (0, \pm a_2, 0, \ldots, 0), \ldots, (0, 0, \ldots, \pm a_2)$ are two axial point sets. Here $(a_1, 0, \ldots, 0)2^1 U (a_2, 0, \ldots, 0)2^1$ indicate the 4v axial points produced from $(a_1, 0, \ldots, 0)$ and $(a_2, 0, \ldots, 0)$ point sets. Let U indicate the union of design points produced from various point sets and central points are represented by $n_0$. Following the methods of [11], [19], [21] we suggest a method of construction on second type SOSRD utilizing SUBA with two unequal block sizes as shown in below theorem.

### 3.1 Theorem (3.1)

$[1-(v, m, r, k_1, k_2, m_1, m_2, \lambda)]2^{t(k)} U (a_1, 0, \ldots, 0)2^1 U (a_2, 0, \ldots, 0)2^1 U (n_0)$, $k = \sup.(k_1, k_2)$ will result in a v-dimensional second type SOSRD utilizing SUBA with two unequal block sizes in $N = \left(m2^{t(k)}\right) + (4v) + n_0$ design points, with the following biquadratic equation

$$[8v-4N](a_1^8+a_2^8)+[16v-8N]a_1^4a_2^4+16v(a_1^6a_2^2+a_1^2a_2^6)+8vr2^{t(k)}(a_1^6+a_1^4a_2^2+a_1^2a_2^4+a_2^6)$$

$$+[-2vN\lambda-4N(r-3\lambda)+4vr+2vr^2\,2^{t(k)}-20v\lambda+16\lambda]2^{t(k)}(a_1^4+a_2^4)+[4vr^2-20rv\lambda+16r\lambda]2^{2t(k)}(a_1^2+a_2^2)$$

$$+[8vr+32\lambda-40v\lambda]2^{t(k)}a_1^2a_2^2+[5v\lambda^2-vr\lambda-(r-3\lambda)^2]N2^{2t(k)}+[vr^3-5v\lambda r^2+4r^2\lambda]2^{3t(k)} = 0 \quad \ldots\ldots\ldots(3.1)$$

The design exists if the equation mentioned above (3.1) contains at least one positive real root.

**Proof.** Regarding the design points produced from SUBA with two unequal block sizes, simple symmetry stipulations A, B, and C of equation (2.2) are true. Since condition (A) of equation (2.2) is obviously true, condition (B) and (C) of (2.2) are also true as follows:

$$\text{B(1).}\quad \sum X_{sw}^{2} = r\,2^{t(k)} + 2\,a_1^2 + 2\,a_2^2 = N\,\delta_2$$

$$\text{(2).}\quad \sum X_{sw}^{4} = r\,2^{t(k)} + 2a_1^4 + 2a_2^4 = c\,N\,\delta_4$$

$$\text{C.}\quad \sum X_{sw}^{2} X_{tw}^{2} = \lambda 2^{t(k)} = N\,\delta_4 \;\ldots\ldots\ldots..(3.2)$$

From B(2) and C of (3.2), we have $c=\frac{r2^{t(k)}+2a_1^4+2a_2^4}{\lambda 2^{t(k)}}$. The result of simplifying equation (2.7) by substituting c, $\delta_2$ and $\delta_4$ is

$$\left[v\left(5-\frac{r2^{t(k)}+2a_1^4+2a_2^4}{\lambda 2^{t(k)}}\right)-\left(\frac{r2^{t(k)}+2a_1^4+2a_2^4}{\lambda 2^{t(k)}}-3\right)^2\right]\frac{\lambda 2^{t(k)}}{N}+\left[v\left(\frac{r2^{t(k)}+2a_1^4+2a_2^4}{\lambda 2^{t(k)}}-5\right)+4\right]\left(\frac{r2^{t(k)}+2a_1^2+2a_2^2}{N}\right)^2=0 \;\ldots\ldots (3.3)$$

The biquadratic equation shown in (3.1) is obtained by simplifying (3.3).

*3.1.1 Example 1*

We demonstrate theorem (3.1) with the construction of second type SOSRD for 12-factors using SUBA with two unequal block sizes $\left(v=12, m=13, r=4, k_1=3, k_2=4, m_1=4, m_2=9, \lambda=1\right)$.

The design points, $[1\text{- }(12, 13, 4, 3, 4, 4, 9, 1)]2^{t(4)}U(a_1,0,\ldots,0)2^1U(a_2,0,\ldots,0)2^1U(n_0=1)$; k=sup(3,4) will result in a second type SOSRD utilizing SUBA with two unequal block sizes in N=257 design points with $n_0=1$ and $a_1=1$.

For the design points produced from second type SOSRD utilizing SUBA with two unequal block sizes, simple symmetry condition (A) of equation (2.2) are true.

Here B and C of equation (3.2) are

$$\text{B.(1)}\quad \sum X_{sw}^{2} = 64+2a_1^2+2a_2^2 = N\delta_2$$

$$\text{(2)}\quad \sum X_{sw}^{4} = 64+2a_1^4+2a_2^4 = cN\delta_4$$

$$\text{C.}\quad \sum X_{sw}^{2} X_{tw}^{2} = 16 = N\delta_4 \qquad \ldots\ldots\ldots\ldots\ldots\ldots..(3.4)$$

From B (2) and C of equation (3.4), we have $c=\frac{64+2a_1^4+2a_2^4}{16}$.

Substitute c, $\delta_2$ and $\delta_4$ in equation (2.7) and on simplifying, we obtain the following biquadratic equation.

$$932(a_1^8+a_2^8)+1864a_1^4a_2^4-192(a_1^6a_2^2+a_1^2a_2^6)-6144(a_1^6+a_1^4a_2^2+a_1^2a_2^4+a_2^6)+17344(a_1^4+a_2^4)+32768(a_1^2+a_2^2)+1024a_1^2a_2^2-199424=0$$

Substitute $a_1=1$ in the above equation and on simplification, we get

$$932a_2^8-6336a_2^6+13064a_2^4+27456a_2^2-154524 = 0. \;\ldots\ldots\ldots\ldots\ldots(3.5)$$

Only a single positive real root exists in equation (3.5) $a_2^2=4.0384 \Rightarrow a_2=2.0096$. The non-singularity criteria D of (2.4) is fulfilled.

In this case, it might be noted that SOSRD second type utilizing SUBA with two unequal block sizes contains 257 design points for 12 factors, while the corresponding second type SOSRD obtained using CCD of Ravikumar and Victorbabu [19], BIBD of Victorbabu and Ravikumar [35], PBD of Ravikumar and Victorbabu [21] for 12 factors need 305, 401 and 561 design points respectively. Hence the suggested method second type SOSRD utilizing SUBA with two unequal block sizes for 12 factors leads to fewer design points than second type SOSRD obtained through CCD, BIBD and PBD respectively.

### *3.1.2 Example 2*

Construction of second type SOSRD for 8-factors using SUBA with two unequal block sizes $\left(v=8, m=12, r=4, k_1=2, k_2=3, m_1=4, m_2=8, \lambda=1\right)$.

The design points, $[1\text{-}(8, 12, 4, 2, 3, 4, 8, 1)]2^{t(3)}U(a_1,0,\ldots,0)2^1U(a_2,0,\ldots,0)2^1U(n_0=1)$; k=sup(2,3) will result in a second type SOSRD utilizing SUBA with two unequal block sizes in N=129 design points with $n_0=1$ and $a_1=1$.

For the design points produced from second type SOSRD utilizing SUBA with two unequal block sizes, simple symmetry condition (A) of equation (2.2) are true.

Here B and C of equation (3.2) are

$$\text{B.(1)} \quad \sum X_{sw}^2=32+2a_1^2+2a_2^2=N\delta_2$$

$$\text{(2)} \quad \sum X_{sw}^4=32+2a_1^4+2a_2^4=cN\delta_4$$

$$\text{C.} \quad \sum X_{sw}^2X_{tw}^2=8=N\delta_4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.6)$$

From B (2) and C of equation (3.6), we have $c=\frac{32+2a_1^4+2a_2^4}{8}$.

Substitute c, $\delta_2$ and $\delta_4$ in equation (2.7) and on simplifying, we obtain the following biquadratic equation.

$$452(a_1^8+a_2^8)+904a_1^4a_2^4-128(a_1^6a_2^2+a_1^2a_2^6)-2048(a_1^6+a_1^4a_2^2+a_1^2a_2^4+a_2^6)+4384(a_1^4+a_2^4)+4096(a_1^2+a_2^2)+256a_1^2a_2^2-25024=0$$

Substitute $a_1=1$ in the above equation and on simplification, we get

$$452a_2^8-2176a_2^6+3240a_2^4+2176a_2^2-18140 = 0. \ldots\ldots\ldots(3.7)$$

Only a single positive real root exists in equation (3.7) $a_2^2=3.3154 \Rightarrow a_2=1.8208$. The non-singularity criteria D of (2.4) is fulfilled (i.e., 0.6248>0.5715).

In this case, it might be noted that second type SOSRD utilizing SUBA with two unequal block sizes contains 129 design points for 8 factors, while the corresponding second type SOSRD obtained using BIBD of Victorbabu and Ravikumar [35], PBD of Ravikumar and Victorbabu [21] for 8 factors need 145 and 273 design points respectively. Hence the suggested method second type SOSRD utilizing SUBA with two unequal block sizes for 8 factors leads to fewer design points than second type SOSRD obtained through BIBD and PBD respectively.

The Appendix of table 1 gives the appropriate second type slope rotatability values of the parameter $a_2$ for designs utilizing SUBA with two unequal block sizes for factors 6≤v≤15.

Table 2 gives the variance of estimated response of the first order partial derivative of second type SOSRD utilizing SUBA with two unequal block sizes for factors 6≤v≤15.

## 4 Conclusion

In this paper second type SOSRD utilizing SUBA with sizes of two different blocks is suggested. It is observed that the suggested method can develop designs containing fewer design points than second type SOSRD obtained utilizing CCD, pairwise balanced designs (PBD) and balanced incomplete block designs (BIBD).

## Compliance with ethical standards

### *Acknowledgement*

The authors are grateful to the referees for their suggestions and also thankful to the editor in chief for accepting the paper for the possible publication in International Journal of Scientific Research Updates.

### *Disclosure of conflict of interest*

No conflict of interest to be disclosed.

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## Appendix

**Table 1** Values of $a_2$ for second type SOSRD utilizing SUBA with two unequal block sizes for 6≤v≤15 with $a_1 = 1$.

[These are second type SOSRDs with design points, $[1\text{- }(v,m,r,k_1,k_2,m_1,m_2,\lambda)]2^{t(k)}U(a_1,0,\ldots,0)2^1U(a_2,0,\ldots,0)2^1U(n_0)]$]

| (6, 7, 3, 2, 3, 3, 4, 1) | | | (6, 11, 7, 3, 4, 2, 9, 4) | | | (6, 15, 9, 2, 4, 3, 12, 5) | | |
|---|---|---|---|---|---|---|---|---|
| **$n_0$** | **N** | **$a_2$** | **$n_0$** | **N** | **$a_2$** | **$n_0$** | **N** | **$a_2$** |
| 1 | 81 | 1.9767 | 1 | 201 | 3.3797 | 1 | 265 | 3.5118 |
| 5 | 85 | 1.8856 | 5 | 205 | 3.3583 | 5 | 269 | 3.4981 |
| 10 | 90 | 1.8035 | 10 | 210 | 3.3348 | 10 | 274 | 3.4825 |
| 15 | 95 | 1.7480 | 15 | 215 | 3.3143 | 15 | 279 | 3.4685 |
| 20 | 100 | 1.7095 | 20 | 220 | 3.2964 | 20 | 284 | 3.4558 |

| (6, 15, 8, 2, 4, 6, 9, 4) | | | (8, 12, 4, 2, 3, 4, 8, 1) | | | (8, 24, 9, 2, 4, 12, 12, 3) | | |
|---|---|---|---|---|---|---|---|---|
| **$n_0$** | **N** | **$a_2$** | **$n_0$** | **N** | **$a_2$** | **$n_0$** | **N** | **$a_2$** |
| 1 | 265 | 3.2611 | 1 | 129 | 1.8208 | 1 | 417 | 2.7911 |
| 5 | 269 | 3.2484 | 5 | 133 | 1.7036 | 5 | 421 | 2.7785 |
| 10 | 274 | 3.2341 | 10 | 138 | 1.5909 | 10 | 426 | 2.7641 |
| 15 | 279 | 3.2211 | 15 | 143 | 1.5127 | 15 | 431 | 2.7508 |
| 20 | 284 | 3.2094 | 20 | 148 | 1.4574 | 20 | 436 | 2.7386 |

| (8, 26, 10, 4, 3, 2, 24, 3) | | | (9, 12, 7, 3, 6, 3, 9, 4) | | | (9, 15, 6, 3, 4, 6, 9, 2) | | |
|---|---|---|---|---|---|---|---|---|
| **$n_0$** | **N** | **$a_2$** | **$n_0$** | **N** | **$a_2$** | **$n_0$** | **N** | **$a_2$** |
| 1 | 449 | 2.9017 | 1 | 421 | 3.8965 | 1 | 277 | 2.6580 |
| 5 | 453 | 2.8795 | 5 | 425 | 3.8876 | 5 | 281 | 2.6236 |
| 10 | 458 | 2.8531 | 10 | 430 | 3.8774 | 10 | 286 | 2.5858 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 15 | 463 | 2.8282 | 15 | 435 | 3.8681 | 15 | 291 | 2.5534 |
| 20 | 468 | 2.8048 | 20 | 440 | 3.8596 | 20 | 296 | 2.5256 |

| **(9, 15, 7, 3, 5, 6, 9, 3)** | | | **(9, 18, 5, 2, 3, 9, 9, 1)** | | | **(9, 30, 9, 2, 3, 9, 21, 2)** | | |
|---|---|---|---|---|---|---|---|---|
| $n_0$ | **N** | $a_2$ | $n_0$ | **N** | $a_2$ | $n_0$ | **N** | $a_2$ |
| 1 | 277 | 2.9756 | 1 | 181 | 1.6214 | 1 | 277 | 2.0828 |
| 5 | 281 | 2.9561 | 5 | 185 | 1.4098 | 5 | 281 | 2.0150 |
| 10 | 286 | 2.9351 | 10 | 190 | 1.1018 | 10 | 286 | 1.9331 |
| 15 | 291 | 2.9173 | | | | 15 | 291 | 1.8566 |
| 20 | 296 | 2.9019 | | | | 20 | 296 | 1.7871 |

| **(10, 11, 5, 4, 5, 5, 6, 2)** | | | **(10, 15, 8, 4, 6, 5, 10, 4)** | | | **(10, 25, 8, 4, 3, 5, 20, 2)** | | |
|---|---|---|---|---|---|---|---|---|
| $n_0$ | **N** | $a_2$ | $n_0$ | **N** | $a_2$ | $n_0$ | **N** | $a_2$ |
| 1 | 217 | 2.7115 | 1 | 521 | 3.8560 | 1 | 441 | 2.5122 |
| 5 | 221 | 2.6739 | 5 | 525 | 3.8459 | 5 | 445 | 2.4748 |
| 10 | 226 | 2.6366 | 10 | 530 | 3.8342 | 10 | 450 | 2.4302 |
| 15 | 231 | 2.6077 | 15 | 535 | 3.8235 | 15 | 455 | 2.3884 |
| 20 | 236 | 2.5851 | 20 | 540 | 3.8138 | 20 | 460 | 2.3496 |

| **(12, 13, 4, 3, 4, 4, 9, 1)** | | | **(12, 15, 7, 4, 6, 3, 12, 3)** | | | **(12, 19, 5, 4, 3, 3, 16, 1)** | | |
|---|---|---|---|---|---|---|---|---|
| $n_0$ | **N** | $a_2$ | $n_0$ | **N** | $a_2$ | $n_0$ | **N** | $a_2$ |
| 1 | 257 | 2.0096 | 1 | 529 | 3.5228 | 1 | 353 | 1.9496 |
| 5 | 261 | 1.9304 | 5 | 533 | 3.5085 | 5 | 357 | 1.8239 |
| 10 | 266 | 1.8567 | 10 | 538 | 3.4925 | 10 | 362 | 1.6553 |
| 15 | 271 | 1.8035 | 15 | 543 | 3.4781 | 15 | 367 | 1.4895 |
| 20 | 276 | 1.7641 | 20 | 548 | 3.4653 | 20 | 372 | 1.3356 |

| **(15, 16, 6, 5, 6, 6, 10, 2)** | | | **(15, 20, 5, 3, 4, 5, 15, 1)** | | | **(15, 33, 9, 5, 4, 3, 30, 2)** | | |
|---|---|---|---|---|---|---|---|---|
| $n_0$ | **N** | $a_2$ | $n_0$ | **N** | $a_2$ | $n_0$ | **N** | $a_2$ |
| 1 | 573 | 3.0527 | 1 | 381 | 1.3611 | 1 | 589 | 2.3498 |
| 5 | 577 | 3.0278 | 5 | 385 | 1.1511 | 5 | 593 | 2.2830 |
| 10 | 582 | 3.0006 | 10 | 390 | 0.8329 | 10 | 598 | 2.2037 |
| 15 | 587 | 2.9774 | | | | 15 | 603 | 2.1318 |
| 20 | 592 | 2.9573 | | | | 20 | 608 | 2.0684 |

**Table 2** The Variance of estimated response of the first order partial derivative of second type SOSRD utilizing SUBA with two unequal block sizes for different factors 6≤v≤15

[These are second type SOSRDs with design points, $\left[1-\left(v,m,r,k_1,k_2,m_1,m_2,\lambda\right)\right]2^{t(k)}U\left(a_1,0,\ldots,0\right)2^1U\left(a_2,0,\ldots,0\right)2^1U\left(n_0\right)$ ]

| $(v,m,r,k_1,k_2,m_1,m_2,\lambda)$ | **N** | **$a_2$** | **$\delta_4$** | **$\delta_2$** | **C** | $\sigma^{-2}V(\hat{\beta}_0)$ | $\sigma^{-2}V(\hat{\beta}_s)$ | $\sigma^{-2}V(\hat{\beta}_{st})$ | $\sigma^{-2}V(\hat{\beta}_{ss})$ | $\sigma^{-2}Cov(\hat{\beta}_0,\hat{\beta}_{ss})$ | $\sigma^{-2}Cov(\hat{\beta}_{ss},\hat{\beta}_{st})$ | $V\left(\frac{\partial \hat{Y}}{\partial X_s}\right)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (6,7,3,2,3,3,4,1) | 81 | 1.9767 | 0.0988 | 0.4175 | 7.0668 | 0.1006 | 0.0296 | 0.1250 | 0.0312 | -0.0352 | 0.0106 | $(0.029570+0.124956d^2)\sigma^2$ |
| (8,12,4,2,3,4,8,1) | 129 | 1.8208 | 0.0620 | 0.3150 | 6.9978 | 0.0908 | 0.0246 | 0.1250 | 0.0313 | -0.0330 | 0.0105 | $(0.024609+0.125031d^2)\sigma^2$ |
| (9,18,5,2,3,9,9,1) | 181 | 1.6214 | 0.0442 | 0.2611 | 6.9778 | 0.0755 | 0.0212 | 0.1250 | 0.0313 | -0.0298 | 0.0103 | $(0.021160+0.124997d^2)\sigma^2$ |
| (10,11,5,4,5,5,6,2) | 217 | 2.7115 | 0.1475 | 0.4456 | 5.9410 | 0.0465 | 0.0103 | 0.0312 | 0.0078 | -0.0094 | 0.0015 | $(0.010342+0.031243d^2)\sigma^2$ |
| (12,13,4,3,4,4,9,1) | 257 | 2.0096 | 0.0623 | 0.2882 | 6.1637 | 0.0573 | 0.0135 | 0.0625 | 0.0156 | -0.0154 | 0.0035 | $(0.013501+0.062457d^2)\sigma^2$ |
| (15,20,5,3,4,5,15,1) | 381 | 1.3611 | 0.0420 | 0.2249 | 5.5540 | 0.0345 | 0.0117 | 0.0625 | 0.0156 | -0.0094 | 0.0019 | $(0.011670+0.062492d^2)\sigma^2$ |

*For all designs we have taken $a_1 = 1$.